

Precision Study of Minimal Supersymmetric Standard Model by production and decay of scalar tau lepton

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Abstract

Study of the production and decay of scalar tau lepton ($\tilde{\tau}$) at future e^+e^- colliders helps to determine the value of $\tan\beta$ through the measurement of the polarization of τ lepton that arises from $\tilde{\tau}$ decay. Key maps of the parameter space of MSSM are presented.

1) Introduction

The Minimal Supersymmetric Standard Model (MSSM)[1] is one of the most promising candidates of the models beyond the Standard Model (SM). It predicts the existence of superpartners of SM particles below a few TeV to remove quadratic divergences which appear in radiative corrections of the SM Higgs sector; thus the model is free from the so-called hierarchy problem of GUT models. It should be noted that the gauge couplings unify very precisely at high energy scale in MSSM [2], consistent with SUSY SU(5) GUT predictions.

Thus searches of SUSY particles at future e^+e^- colliders would be one of its important physics targets. Furthermore, a highly polarized electron beam available for the future linear colliders reduces the background from W^+W^- pair production to the SUSY signals drastically, making it possible to study SUSY parameters very precisely [3]. It was also demonstrated that some SUSY parameters, such as masses and couplings of SUSY particles can be measured very precisely by studying the production and decay of the first and second generation of sleptons ($\tilde{e}, \tilde{\mu}$) and the lighter chargino (χ_1^+)[3, 4]. The precise measurements of those parameters would give big impacts to the supergravity models and superstring models, which predict relations between various soft SUSY breaking parameters at the Planck scale[5].

In this talk, I would like to talk about the study of the production and the decay of the scalar tau ($\tilde{\tau}$), which were not studied previously. The mode turns out to contain novel information about the tau Yukawa coupling Y_τ or $\tan\beta$ [6], which is very difficult to determine by studying other modes.

$\tilde{\tau}$ production and decay is different from that of \tilde{e} and $\tilde{\mu}$ because the (scalar) tau lepton has a non-negligible Yukawa coupling $Y_\tau \propto m_\tau / \cos\beta$; the coupling would be enhanced linearly to $\tan\beta$ for large value of $\tan\beta$. A consequence of the non-negligible Yukawa coupling is existence

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of a left-right mixing of $\tilde{\tau}$; this suggests that the lighter mass eigenstate of $\tilde{\tau}$ would be lighter than \tilde{e} or $\tilde{\mu}$.

The same Yukawa coupling appears as a non-negligible $\tau\tilde{\tau}\tilde{H}_1^0$ coupling, where \tilde{H}_1^0 is a neutral higgsino. This interaction is involved in $\tilde{\tau}$ decay into a neutralino χ_i^0 and τ , since the χ 's are mixtures of higgsinos and gauginos. Another feature of $\tilde{\tau}$ decay that distinguishes it from other slepton decays is that the τ lepton arising from the decay $\tilde{\tau} \rightarrow \chi_i^0\tau$ decays further in the detector, which enables us to measure the average polarization of the τ (P_τ) [7, 8, 9]. One can then determine a combination of the higgsino-gaugino mixing of χ_i^0 and $\tan\beta$ by measuring both the cross section for $\tilde{\tau}$ production and the $P_\tau(\tilde{\tau} \rightarrow \chi_i^0\tau)$. Especially the sensitivity of P_τ to $\tan\beta$ helps us to determine $\tan\beta(> 5)$, by combining the information from the other mode.

2) The Model

To be more specific, we describe the SUSY parameters that appear in the MSSM. In this model, the Higgs sector consists of two $SU(2)$ doublets, H_1 and H_2 , and the coupling to the matter sector is described by the superpotential

$$W = Y_l H_1 \cdot L E^c + Y_d H_1 \cdot Q D^c + Y_u H_2 \cdot Q U^c. \quad (1)$$

Here E , D , and U are $SU(2)$ singlet lepton and quark superfields, while L and Q are $SU(2)$ doublet sfermion superfields respectively. Both of the neutral components of Higgs doublets (H_1^0 , H_2^0) would have vacuum expectation values and we define $\tan\beta = \langle H_1^0 \rangle / \langle H_2^0 \rangle$. Yukawa couplings Y are represented by β as $Y_{\tau(b)} = gm_{\tau(b)} / (\sqrt{2}m_W \cos\beta)$ and $Y_t = gm_t / (\sqrt{2}m_W \sin\beta)$ respectively. It should be noted that $Y_{\tau(b)}$ is not negligible for large value of $\tan\beta$.

Superpartners of higgsinos and gauginos mix due to the $SU(2) \times U(1)$ symmetry breaking. Its neutral and charged mass eigenstates are called neutralinos χ_i^0 ($i = 1, 2, 3, 4$) and charginos χ_i^{\pm} ($i = 1, 2$), and their mass matrices are described as follows;

$$\begin{aligned} \mathcal{M}_N(\tilde{B}, \tilde{W}_3, \tilde{H}_1^0, \tilde{H}_2^0) = & \\ \begin{pmatrix} M_1 & 0 & -m_Z \sin\theta_W \cos\beta & m_Z \sin\theta_W \sin\beta \\ 0 & M_2 & m_Z \cos\theta_W \cos\beta & -m_Z \cos\theta_W \sin\beta \\ -m_Z \sin\theta_W \cos\beta & m_Z \cos\theta_W \cos\beta & 0 & -\mu \\ m_Z \sin\theta_W \sin\beta & -m_Z \cos\theta_W \sin\beta & -\mu & 0 \end{pmatrix}, & \end{aligned} \quad (2a)$$

$$\mathcal{M}_C(\tilde{W}, \tilde{H}) = \begin{pmatrix} M_2 & m_W \sqrt{2} \sin\beta \\ m_W \sqrt{2} \cos\beta & \mu \end{pmatrix}. \quad (2b)$$

These mass matrices are diagonalized by a real orthogonal matrix N for \mathcal{M}_N , and unitary matrices U and V for \mathcal{M}_C as follows:

$$U^* \mathcal{M}_C V^{-1} = M_D^C, \quad N \mathcal{M}_N N^{-1} = M_D^N. \quad (3)$$

Here M_1 and M_2 are soft breaking gaugino mass parameters of \tilde{B} and \tilde{W} , while μ is a supersymmetric Higgsino mass parameter.

Due to the R-parity conservation of MSSM and some cosmological constraint, the lightest neutralino χ_1^0 is likely the lightest SUSY particle (LSP) and stable, thus escapes from detection at collider experiments. We assume this throughout the discussions of this paper.

Left and right scalar fermions also mix due to the $SU(2) \times U(1)$ symmetry breaking. However, the mixing is negligible for the first and the second generation sfermions. The mixing of the third generation sfermions would be described by the mass matrix of scalar tau lepton $\tilde{\tau}_{L(R)}$ as

$$\mathcal{M}_{\tilde{\tau}}^2 = \begin{pmatrix} m_{LL}^2 & m_{LR}^2 \\ m_{LR}^2 & m_{RR}^2 \end{pmatrix} = \begin{pmatrix} m_L^2 + m_{\tau}^2 + 0.27D & -m_{\tau}(A_{\tau} + \mu \tan \beta) \\ -m_{\tau}(A_{\tau} + \mu \tan \beta) & m_R^2 + m_{\tau}^2 + 0.23D \end{pmatrix} \cdot \begin{pmatrix} \tilde{\tau}_L \\ \tilde{\tau}_R \end{pmatrix} \quad (4)$$

where m_R and m_L are soft breaking scalar mass parameters of $\tilde{\tau}_R$ and $(\tilde{\nu}_{\tau}, \tilde{\tau})_L$, A_{τ} is a trilinear coupling of $\tilde{\tau}_L \tilde{\tau}_R H_1$ and $D = -m_Z^2 \cos(2\beta)$. $\tilde{\tau}_R$ and $\tilde{\tau}_L$ then mix to form two mass eigenstates $\tilde{\tau}_1$ and $\tilde{\tau}_2$ ($m_{\tilde{\tau}_1} < m_{\tilde{\tau}_2}$)

$$\begin{pmatrix} \tilde{\tau}_1 \\ \tilde{\tau}_2 \end{pmatrix} = \begin{pmatrix} \cos \theta_{\tau} & \sin \theta_{\tau} \\ -\sin \theta_{\tau} & \cos \theta_{\tau} \end{pmatrix} \begin{pmatrix} \tilde{\tau}_L \\ \tilde{\tau}_R \end{pmatrix}. \quad (5)$$

Notice m_{LR}^2 could be large for very large value of $\tan \beta(\mu)$, leading $m_{\tilde{\tau}_1}$ smaller than m_{LL} or m_{RR} . In the models which predict the equal scalar masses at GUT scale such as the minimal supergravity model or the superstring model, $m_{\tilde{\tau}}$ can be lighter than $m_{\tilde{e}}$ or $m_{\tilde{\mu}}$. This is because both by the $\tilde{\tau}$ mixing and also the effect of the negative RG running of $m_{L(R)\tau}$ by τ Yukawa coupling which makes $m_{L(R)\tau}$ is lighter than those of \tilde{e} and $\tilde{\mu}$ at the weak scale. $\tilde{\tau}$ analysis is important in the sense that it might be found earlier than the other sfermions in future collider experiments.

3) $\tilde{\tau}$ decay

It was demonstrated in Ref.[3] that some of the above mass parameters could be determined precisely by proposed linear colliders with a highly polarized electron beam. The masses of the lightest neutralino, the lighter chargino, the selectron and the smuon were shown to be determined with an error of a few % for a representative parameter by the energy distribution of leptons or jets coming from decaying sparticles. Furthermore, by measuring other quantities

such as the production cross section of selectron, the gaugino mass parameter $M_{1(2)}$ was determined also with an error of a few %. In the paper, it has also been discussed that SUGRA GUT relations such as $m_{\tilde{e}} = m_{\tilde{\mu}}$ and $M_1/M_2 = \frac{5}{3} \tan^2 \theta_W$ would be checked with comparable precision.

No analysis in this direction has been made for $\tilde{\tau}$ production previously. This is because, for one thing, the mode is not easy to analyze as the τ leptons which arise from the decay $\tilde{\tau}_1 \rightarrow \tau \chi_i^0$ further decay inside the detector, thus the kinematics is not easy compared to the modes previously studied. However, it has been pointed out in [6], the fact that τ lepton decays further gives an interesting opportunity to measure the polarization of the τ lepton (P_τ). The polarization is directly related to the value of $\tan \beta$, as we discuss below.

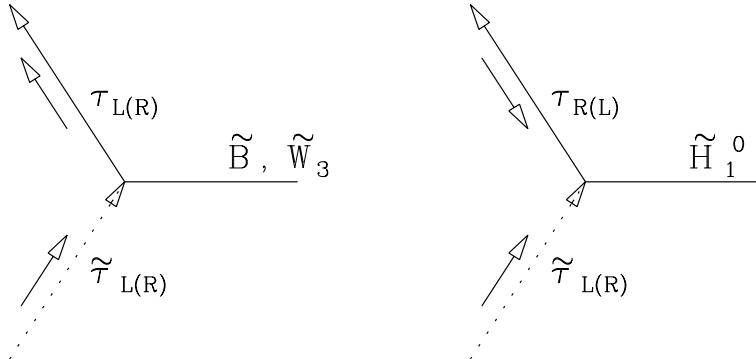


Figure 1:

Fig.1 shows the interaction of neutral components of gauginos and higgsinos to $\tilde{\tau}$ and τ . The interaction is completely fixed by the gauge and supersymmetry of the model. The coupling to the gaugino $\tilde{B}(\tilde{W}_3)$ is proportional to the gauge coupling $g_{1(2)}$, while the coupling of the $\tilde{\tau}$ to the Higgino \tilde{H}_1^0 is proportional to Y_τ . Not only the couplings, the two interactions are different if one looks into the chirality of the (s)fermion. The (super-) gauge interaction is chirality conserving, while the (super-) Yukawa interaction flips the chirality. (In the figure, the arrows next to the $\tilde{\tau}$ and τ line show the direction of chirality.) Thus the P_τ depends on the ratio of the chirality flipping and the conserving interactions.

As we mentioned already, gauginos and higgsinos are not mass eigenstates, but they mix to form neutralino mass eigenstates χ_i^0 . $\tilde{\tau}_R$ and $\tilde{\tau}_L$ also mix, thus the coupling of the $\tilde{\tau}_1 \tau \chi_1^0$ interaction depends on both the stau mixing θ_τ and the neutralino mixing N_{ij} . However, the dependence of θ_τ would be removed by measuring the production cross section of $\tilde{\tau}$ precisely,

as $\sigma(e^+e^- \rightarrow \tilde{\tau}^+\tilde{\tau}^-)$ depends on θ_τ only. *

For an illustrative purpose, let's discuss the case where $\tilde{\tau}_1 = \tilde{\tau}_R$. Then $P_\tau(\tilde{\tau}_R \rightarrow \chi_i^0 \tau)$ is expressed, in the limit that the final state τ is relativistic, as follows;

$$P_\tau(\tilde{\tau}_R \rightarrow \chi_i^0 \tau) = \frac{(\sqrt{2}gN_{11} \tan\theta_W)^2 - (Y_\tau N_{13})^2}{(\sqrt{2}gN_{11} \tan\theta_W)^2 + (Y_\tau N_{13})^2}. \quad (6)$$

Here N_{ij} is a neutralino diagonalization matrix appears in Eq.(3) and Y_τ is the tau collaborations coupling in Eq.(1).

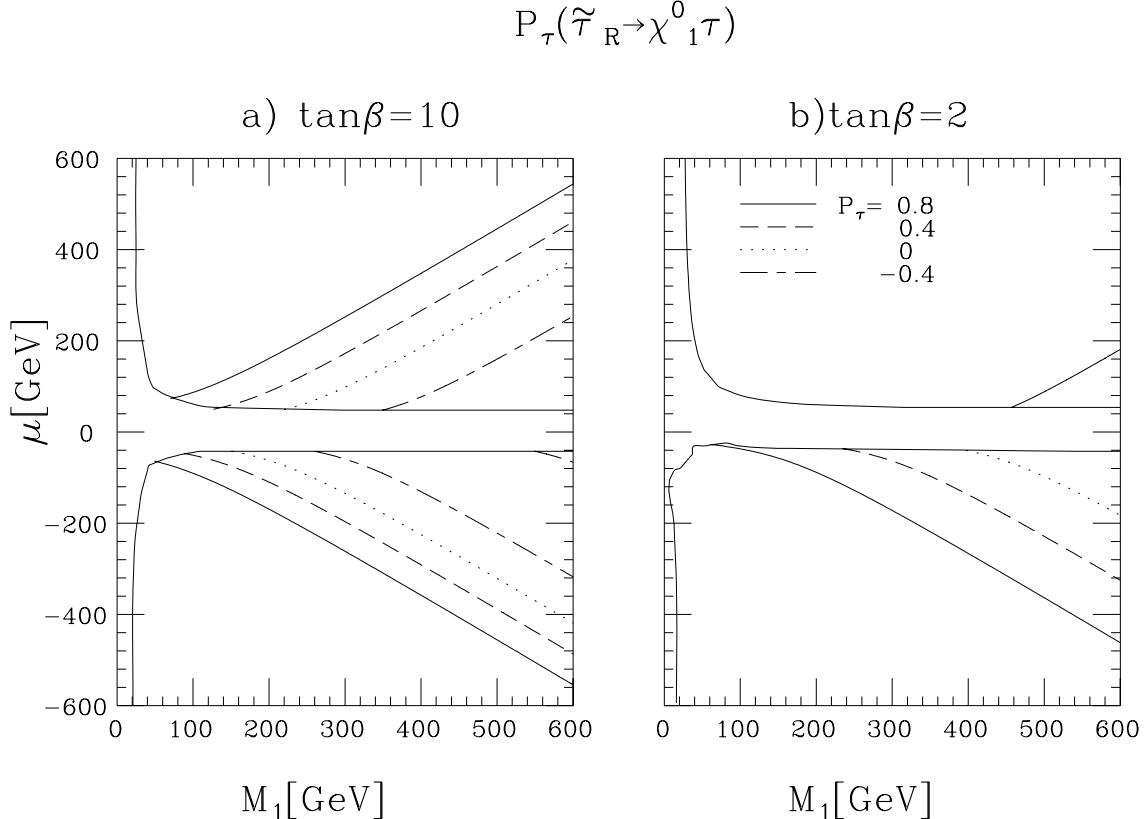


Figure 2:

*The measurement of the σ is actually correlated to the mixing of neutralino, as the detection efficiency depends on the decay modes into $\chi_i^0 \tau$, which should be studied carefully. The detailed discussion can be found in [6]

In fig. 2 we show contours of constant $P_\tau(\tilde{\tau}_R \rightarrow \chi_1^0 \tau)$ in the $M_1 - \mu$ plane for $\tan\beta = 10$ (Fig. 2a) and $\tan\beta = 2$ (Fig. 2b). P_τ decreases monotonically as M_1 increases for a fixed value of μ , as χ_1^0 is bino-like ($N_{11} \simeq 1$) for $M_1 \ll |\mu|$, while χ_1^0 is higgsino-like if $M_1 \gg |\mu|$ ($N_{11} \ll 1$). One would also notice that strong dependence of P_τ on $\tan\beta$. This is because the Yukawa coupling of the τ lepton increases linearly with $\tan\beta$ when $\tan\beta$ is large.

We learned that the measurement of P_τ gives us a constraint to a combination of neutralino mixing N_{ij} and $\tan\beta$. Other sparticle productions also carry information about the neutralino sector. However as $\tan\beta$ becomes larger, the neutralino and chargino mass and mixing matrix become less and less dependent on $\tan\beta$. This is because the off-diagonal elements of the mass matrices of the neutralinos and charginos become insensitive to $\tan\beta$ once $\tan\beta > 10$, as $\cos\beta \sim 1$ and $\sin\beta \sim 0$.

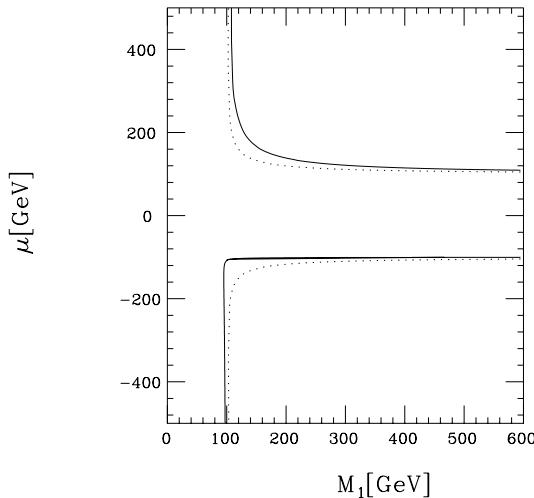


Figure 3:

To demonstrate this, we show various quantities in Fig.4 a)-d) fixing $m_{\chi_1^0} = 100$ GeV, and varying M_1 and $\tan\beta$. The reasons to take such a parametrization are following: According to the investigation of the production $e_R^+ \tilde{e}_R^- (\tilde{\mu}_R^+ \tilde{\mu}_R^-)$ and its subsequent decay to $e^\pm(\mu^\pm)\chi_1^0$ in Ref[3], one can determine the neutralino mass with an error of a few percent from the distribution of the final state leptons. The same analysis is also possible for the chargino pair production and decay, so it would be almost certain that we get some idea of the lightest neutralino mass $m_{\chi_1^0}$ once SUSY particle productions are observed at an e^+e^- collider. The corresponding curves that satisfy the constraint were shown in Fig. 3 on $M_1 - \mu$ plain, assuming $m_{\chi_1^0} = 100$ GeV (no error). Here the solid curve corresponds to $\tan\beta = 1.5$ and the dotted curve

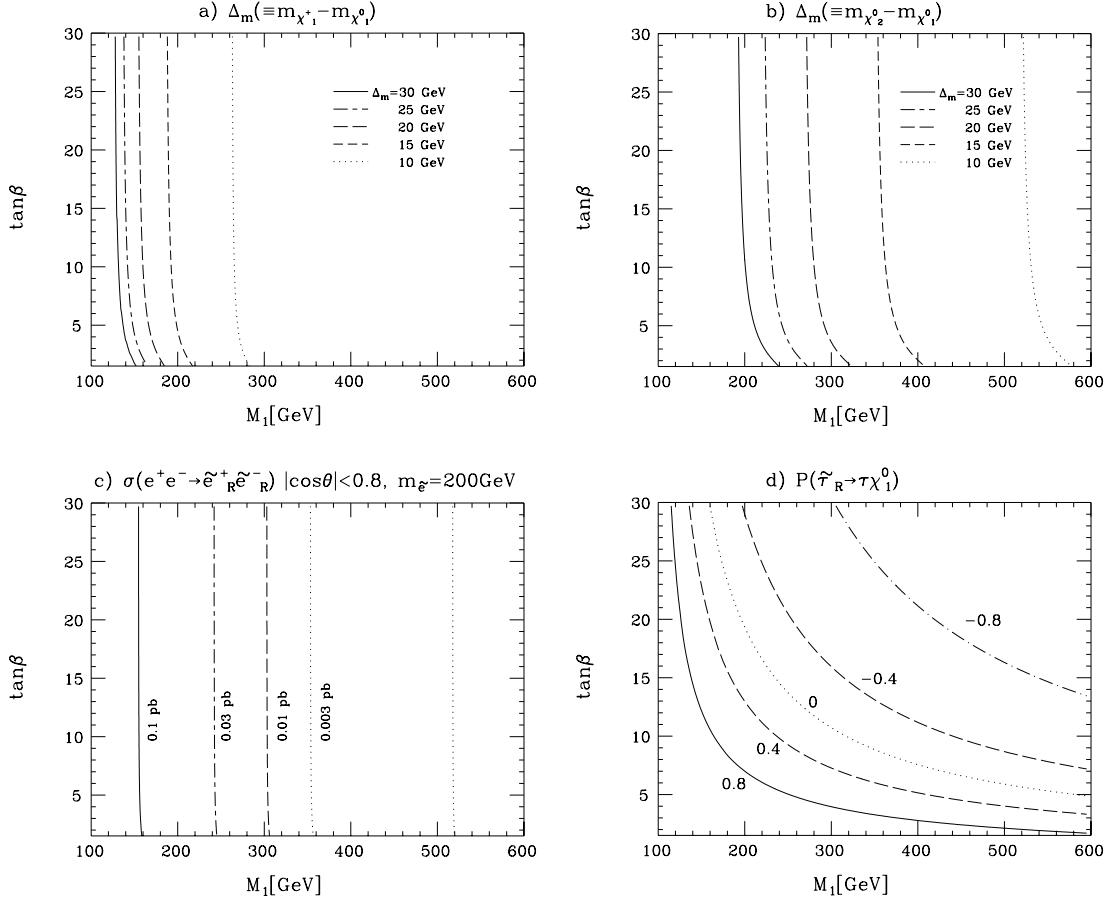


Figure 4:

corresponds to $\tan \beta = 30$. [†] With the mass constraint, one can specify the parameter space of the neutralino sector by M_1 and $\tan \beta$, up to two ambiguities of positive and negative μ solutions for the large M_1 and $\tan \beta$ region, or up to 2 solutions in the negative μ region and one solution in the positive μ region for the small values of M_1 and $\tan \beta$. We take the positive μ solution throughout the plots Fig. 4 a)-d) to avoid too many lines (sometimes quite close each other) appearing on the same plot. [‡]

Fig. 4a and fig. 4b show the contours of the mass differences a) $m_{\chi_1^+} - m_{\chi_1^0}$ and b) $m_{\chi_2^0} - m_{\chi_1^0}$. One can see the mass difference depends very mildly on $\tan \beta$ once $\tan \beta > 5$. The tendency is

[†]We assumed GUT relation to the gaugino masses

[‡]The ambiguity of the μ might be removed by other experiment, such as $Br(b \rightarrow s\gamma)$ [10]

also same for the negative μ solution, though the mass differences decreases as $\tan\beta$ becomes smaller for the negative μ solution. The dependence on $\tan\beta$ is even milder for $\sigma(e^+e^- \rightarrow \tilde{e}_R^+\tilde{e}_R^-)$ as can be seen in Fig 4c. The production proceeds through the s-channel exchange of gauge bosons and the t-channel exchange of neutralinos, where the dependence on M_1 comes in. The production cross section turns out to be the best quantity to fix M_1 , free from the uncertainty of the value of $\tan\beta$.

Fig. 4d) is the contour plot of constant P_τ ($\tilde{\tau}_R \rightarrow \chi_1^0\tau$). The plot looks totally different from Fig 4a-c). If M_1 is not too much close to 100 GeV, the polarization depends on $\tan\beta$ sensitively for the parameters shown in the figure. If one knows M_1 precisely from the production cross section of \tilde{e}_R or from other processes, one can extract $\tan\beta$ by further measuring $P_\tau(\tilde{\tau}_R \rightarrow \chi_1^0\tau)$. Notice that the sensitivity is better in the region $\tan\beta > 5$, complimentary to the information from the mass differences of inos. (See Fig 4a and 4b).

For most of the parameter space, $m_{\chi_1^0}$, $m_{\chi_2^0}$, $m_{\chi_1^+}$ are very close to each other, thus the decay mode into those inos are always open. The determination of the branching ratios constrain the model parameters even further.

Notice the decay into the lightest neutralino may not be a dominant decay mode. If the decay modes into the gaugino like ino are open, $\tilde{\tau}$ decays dominantly into the ino, even if the higgsino-like ino is lighter than the gaugino-like ino. As the gaugino coupling is insensitive to $\tan\beta$, we will not be able to determine $\tan\beta$ in such a case.

4) Measurement of P_τ

The measurement of P_τ would be carried out through the energy distribution of decay products from the polarized τ lepton. The τ lepton decays into $A\nu_\tau$ where $A = e\nu, \pi, \rho, a_1, \dots$. The decay distributions of the τ decay products depend on the polarization of the parent τ lepton[10]. In particular, for each decay channel the momentum distribution of the decay products (π^- , $\rho \rightarrow \pi^-\pi^0, \dots$) differs significantly depending on whether their parent is τ_R^- ($h = 1/2$) or τ_L^- ($h = -1/2$). If the τ lepton is relativistic, P_τ can then be determined from the energy distribution of the decay products [8]. Notice that the τ lepton from a $\tilde{\tau}$ decay also has some energy distribution which depends on $m_{\tilde{\tau}_1}$ and $m_{\chi_1^0}$, thus the energy spectrum of the final decay products depends on $m_{\tilde{\tau}_1}$, $m_{\chi_1^0}$ and P_τ . The three quantity, in principle, can be determined from the energy distribution only by fitting the energy spectrum, though experimentally rather challenging. The situation can be improved by using information from other sources: $m_{\chi_1^0}$ from, for instance, selectron production and decay, and $m_{\tilde{\tau}_1}$ from threshold scan.

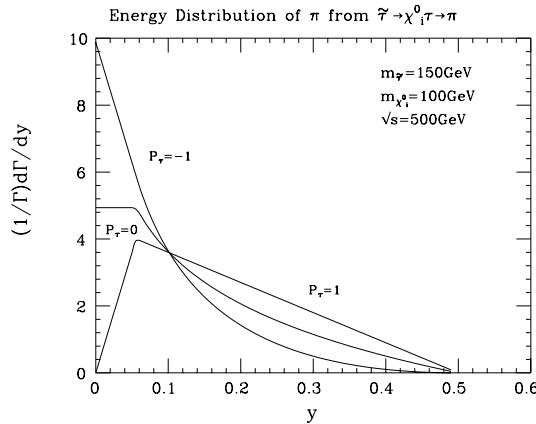


Figure 5:

In fig. 5 we have shown the normalized energy distribution of the π ($y = E_\pi/E_{\text{beam}}$) from the cascade decay of $\tilde{\tau}_1$. For the plot we took $P_\tau = \pm 1, 0$, $m_{\tilde{\tau}_1} = 150$ GeV, $m_{\chi_1^0} = 100$ GeV and $\sqrt{s} = 500$ GeV. Reflecting the hard (soft) spectrum of the π from $\tau_{R(L)}$, the spectrum is considerably harder (softer) for $\tau_{R(L)}$. The upper end of the energy distribution (y_{max}) is the maximum energy of the τ lepton arising from $\tilde{\tau}$, while the minimum energy of the τ lepton corresponds to the peak of the energy distribution for $P_\tau = 1$ (y_{min}). For the parameter used in Fig 5, 52%(90%) of the events are in $y > y_{\text{min}}$ region for $P_\tau = -1(1)$, while 4% (22%) of the events go above $(y_{\text{min}} + y_{\text{max}})/2$.

P_τ can also be measured independently by studying the distributions of the difference of the energy between decay pions from ρ and a_1 . The $\tau \rightarrow \rho\nu$ mode has a branching ratio of about 23% and the $\tau \rightarrow a_1\nu$ mode has a branching ratio of about 15%. $\tau_{L(R)}$ decays dominantly into the longitudinal (transverse) element of ρ or a_1 , and they tend to get most of the τ energy. Then, a transversely polarized ρ favors equal splitting of the ρ energy between the two decay pions, whereas a longitudinally polarized ρ leads to a large difference of the π^- and π^0 energies. For a_{1T} , all three pions have a tendency to share equally the energy of a_1 . On the other hand, a_{1L} again favors configurations in which one or two of the pions are soft. A detailed discussion of the energy distribution may be found in [8].

Monte Carlo simulations of the τ lepton polarization are in progress [11]. For this purpose, we have developed a Monte Carlo event generator of the signal process ($e^+e^- \rightarrow \tilde{\tau}^+\tilde{\tau}^-$). The generator takes into account the polarization of decaying taus, by using the TAUOLA2.4 program package[9]. This package is implemented together with the JETSET7.3 program in a single program module dealing with the hadronization step and is being used for both the signal and background processes. The generated events are processed through the standard JLC detector simulator whose parameters can be found in [12].

As an example to demonstrate the effects of the tau polarization in the stau decays, we show in Fig.6 the momentum fraction distributions of π 's from a_1 decays after a set of cuts to select 1-3 topology.

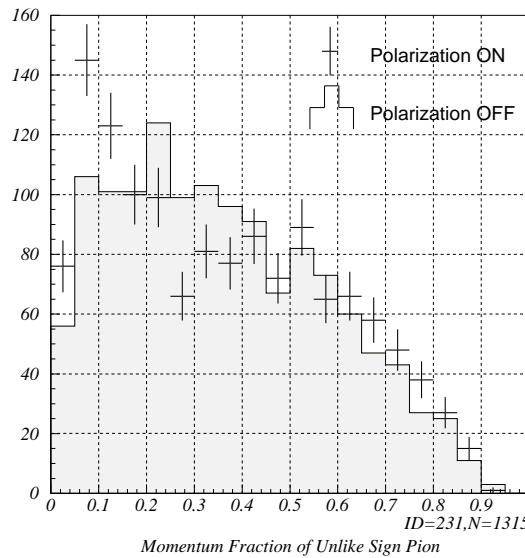


Figure 6:

The original sample contains 5k $\tilde{\tau}$ -pair events in which one tau was forced to decay into $\pi\nu_\tau$ and the other into $a_1\nu_\tau$. The background processes such as W^+W^- , ZZ , ZH , $\gamma\gamma$, etc are not included yet.

We can see that the most sensitive to the τ polarization is the distribution of the energy normalized by the a_1 momentum of the most energetic π 's.

Studies of the other decay modes are on going together with the background processes.

As summary, the measurement of $P_\tau(\tilde{\tau} \rightarrow \tau\chi_i^0)$ would give the unique information about $\tan\beta$, combined with the the information from other measurements such as $m_{\chi_2^0} - m_{\chi_1^0}$ or $\sigma(e^+e^- \rightarrow \tilde{e}^+\tilde{e}^-)$. In some sense Fig 4 might be regarded as key maps of the parameter space of MSSM. After years of running of JLC, we may put our finger on a point of the parameter space by the precise measurement of event signatures of sparticle productions.

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Abstract

Study of the production and decay of scalar tau lepton ($\tilde{\tau}$) at future e^+e^- colliders helps to determine the value of $\tan\beta$ through the measurement of the polarization of τ lepton that arises from $\tilde{\tau}$ decay. Key maps of the parameter space of MSSM are presented.

1) Introduction

The Minimal Supersymmetric Standard Model (MSSM)[1] is one of the most promising candidates of the models beyond the Standard Model (SM). It predicts the existence of superpartners of SM particles below a few TeV to remove quadratic divergences which appear in radiative corrections of the SM Higgs sector; thus the model is free from the so-called hierarchy problem of GUT models. It should be noted that the gauge couplings unify very precisely at high energy scale in MSSM [2], consistent with SUSY SU(5) GUT predictions.

Thus searches of SUSY particles at future e^+e^- colliders would be one of its important physics targets. Furthermore, a highly polarized electron beam available for the future linear colliders reduces the background from W^+W^- pair production to the SUSY signals drastically, making it possible to study SUSY parameters very precisely [3]. It was also demonstrated that some SUSY parameters, such as masses and couplings of SUSY particles can be measured very precisely by studying the production and decay of the first and second generation of sleptons ($\tilde{e}, \tilde{\mu}$) and the lighter chargino (χ_1^+)[3, 4]. The precise measurements of those parameters would

*E-mail: nojirim@theory.kek.jp. talk at the 5th workshop on Japan Linear Collider (JLC) at Kawatabi (Feb. 16-17)

severely constrain supergravity and superstring models, which predict relations between various soft SUSY breaking parameters at the Planck scale[5].

In this talk, I would like to report on a new study of the production and the decay of the scalar tau ($\tilde{\tau}$). This channel turns out to contain novel information about the tau Yukawa coupling Y_τ or $\tan\beta$ [6], which is very difficult to determine by studying other modes.

$\tilde{\tau}$ production and decay is different from that of \tilde{e} and $\tilde{\mu}$ because the (scalar) tau lepton has a non-negligible Yukawa coupling $Y_\tau \propto m_\tau / \cos\beta$; the coupling would be enhanced linearly $\propto \tan\beta$ for large value of $\tan\beta$.

A consequence of the non-negligible Yukawa coupling is existence of left-right mixing of $\tilde{\tau}$. The lighter mass eigenstate of $\tilde{\tau}$ would be lighter than \tilde{e} or $\tilde{\mu}$, even if mass parameter of $\tilde{\tau}$ is equal to that of \tilde{e} and $\tilde{\mu}$. This will be discussed briefly in section 2.

The same Yukawa coupling appears as a non-negligible $\tau\tilde{\tau}H_1^0$ coupling, where H_1^0 is a neutral higgsino. This interaction is involved in $\tilde{\tau}$ decay into a neutralino χ_i^0 and τ , since the χ 's are mixtures of higgsinos and gauginos. Another feature of $\tilde{\tau}$ decay that distinguishes it from other slepton decays is that the τ lepton arising from the decay $\tilde{\tau} \rightarrow \chi_i^0\tau$ decays further in the detector, which enables us to measure the average polarization of the τ (P_τ) [7, 8, 9]. One can then determine a combination of the higgsino-gaugino mixing of χ_i^0 and $\tan\beta$ by measuring both the cross section for $\tilde{\tau}$ production and the $P_\tau(\tilde{\tau} \rightarrow \chi_i^0\tau)$. Especially the sensitivity of P_τ to $\tan\beta$ helps us to determine $\tan\beta(> 5)$, by combining the information from the other mode. In section 3 and 4 we are going to discuss this in some detail.

2) The Model

To be more specific, we describe the SUSY parameters that appear in the MSSM. In this model, the Higgs sector consists of two $SU(2)$ doublets, H_1 and H_2 , and the coupling to the matter sector is described by the superpotential

$$W = Y_l H_1 \cdot L E^c + Y_d H_1 \cdot Q D^c + Y_u H_2 \cdot Q U^c. \quad (1)$$

Here E , D , and U are $SU(2)$ singlet lepton and quark superfields, while L and Q are $SU(2)$ doublet sfermion superfields respectively. Both of the neutral components of Higgs doublets (H_1^0 , H_2^0) would have vacuum expectation values and we define $\tan\beta = \langle H_1^0 \rangle / \langle H_2^0 \rangle$. Yukawa couplings Y are related to β as $Y_{\tau(b)} = g m_{\tau(b)} / (\sqrt{2} m_W \cos\beta)$ and $Y_t = g m_t / (\sqrt{2} m_W \sin\beta)$ respectively. It should be noted that $Y_{\tau(b)}$ is not negligible for large value of $\tan\beta$.

Superpartners of higgsinos and gauginos mix due to $SU(2) \times U(1)$ symmetry breaking. Its neutral and charged mass eigenstates are called neutralinos χ_i^0 ($i = 1, 2, 3, 4$) and charginos χ_i^{\pm} ($i = 1, 2$), and their mass matrices are described as follows;

$$\mathcal{M}_N(\tilde{B}, \tilde{W}_3, \tilde{H}_1^0, \tilde{H}_2^0) = \begin{pmatrix} M_1 & 0 & -m_Z \sin\theta_W \cos\beta & m_Z \sin\theta_W \sin\beta \\ 0 & M_2 & m_Z \cos\theta_W \cos\beta & -m_Z \cos\theta_W \sin\beta \\ -m_Z \sin\theta_W \cos\beta & m_Z \cos\theta_W \cos\beta & 0 & -\mu \\ m_Z \sin\theta_W \sin\beta & -m_Z \cos\theta_W \sin\beta & -\mu & 0 \end{pmatrix}, \quad (2a)$$

$$\mathcal{M}_C(\tilde{W}, \tilde{H}) = \begin{pmatrix} M_2 & m_W \sqrt{2} \sin\beta \\ m_W \sqrt{2} \cos\beta & \mu \end{pmatrix}. \quad (2b)$$

Here M_1 and M_2 are soft breaking gaugino mass parameters of \tilde{B} and \tilde{W} , while μ is a supersymmetric Higgsino mass parameter. These mass matrices are diagonalized by a real orthogonal matrix N for \mathcal{M}_N , and unitary matrices U and V for \mathcal{M}_C as follows:

$$U^* \mathcal{M}_C V^{-1} = M_D^C, \quad N \mathcal{M}_N N^{-1} = M_D^N. \quad (3)$$

Due to the R-parity conservation of MSSM and some cosmological constraint, the lightest neutralino χ_1^0 is likely the lightest SUSY particle (LSP) and stable, thus escapes from detection at collider experiments. We assume this throughout the discussions of this paper.

Left and right scalar fermions also mix due to the $SU(2) \times U(1)$ symmetry breaking. However, the mixing is negligible for the first and the second generation sfermions. The mass matrix of scalar tau lepton $\tilde{\tau}_{L(R)}$ would be described as

$$\mathcal{M}_{\tilde{\tau}}^2 = \begin{pmatrix} m_{LL}^2 & m_{LR}^2 \\ m_{LR}^2 & m_{RR}^2 \end{pmatrix} = \begin{pmatrix} m_L^2 + m_{\tau}^2 + 0.27D & -m_{\tau}(A_{\tau} + \mu \tan\beta) \\ -m_{\tau}(A_{\tau} + \mu \tan\beta) & m_R^2 + m_{\tau}^2 + 0.23D \end{pmatrix} \cdot \begin{pmatrix} \tilde{\tau}_L \\ \tilde{\tau}_R \end{pmatrix} \quad (4)$$

where m_R and m_L are soft breaking scalar mass parameters of $\tilde{\tau}_R$ and $(\tilde{\nu}_{\tau}, \tilde{\tau})_L$, A_{τ} is a trilinear coupling of $\tilde{\tau}_L \tilde{\tau}_R H_1$ and $D = -m_Z^2 \cos(2\beta)$. $\tilde{\tau}_R$ and $\tilde{\tau}_L$ then mix to form two mass eigenstates $\tilde{\tau}_1$ and $\tilde{\tau}_2$ ($m_{\tilde{\tau}_1} < m_{\tilde{\tau}_2}$)

$$\begin{pmatrix} \tilde{\tau}_1 \\ \tilde{\tau}_2 \end{pmatrix} = \begin{pmatrix} \cos\theta_{\tau} & \sin\theta_{\tau} \\ -\sin\theta_{\tau} & \cos\theta_{\tau} \end{pmatrix} \begin{pmatrix} \tilde{\tau}_L \\ \tilde{\tau}_R \end{pmatrix}. \quad (5)$$

Notice m_{LR}^2 could be large for very large value of $\tan\beta(\mu)$, so that $m_{\tilde{\tau}_1}$ is smaller than m_{LL} or m_{RR} . In the models which predict the equal scalar masses at GUT scale such as the minimal supergravity model or the superstring model, $m_{\tilde{\tau}}$ can be lighter than $m_{\tilde{e}}$ or $m_{\tilde{\mu}}$. This is because of the $\tilde{\tau}$ mixing and also the effect of the negative RG running of $m_{LL(RR)}$ of $\tilde{\tau}$ by τ Yukawa coupling which makes the mass parameter smaller than those of \tilde{e} and $\tilde{\mu}$ at the weak scale. $\tilde{\tau}$ analysis is important in the sense that it might be found earlier than the other sfermions in future collider experiments.

3) $\tilde{\tau}$ decay

It was demonstrated in Ref.[3] that some of the above mass parameters can be determined precisely by proposed linear colliders with a highly polarized electron beam. The masses of the lightest neutralino, the lighter chargino, the selectron and the smuon were shown to be determined with an error of a few % for a representative parameter set by the energy distribution of leptons or jets coming from decaying sparticles. Furthermore, by measuring other quantities such as the production cross section of selectron, the gaugino mass parameter $M_{1(2)}$ was determined also with an error of a few %. In their paper, it has also been shown that SUGRA GUT relations such as $m_{\tilde{e}} = m_{\tilde{\mu}}$ and $M_1/M_2 = \frac{5}{3} \tan^2 \theta_W$ would be checked with comparable precision.

No analysis in this direction has been made for $\tilde{\tau}$ production previously. This is because, for one thing, the mode is not easy to analyze as the τ leptons which arise from the decay $\tilde{\tau}_1 \rightarrow \tau \chi_i^0$ further decay inside the detector, thus the kinematics is not easy compared to the modes previously studied. However, as has been pointed out in [6], the fact that τ lepton decays further gives an interesting opportunity to measure the polarization of the τ lepton (P_τ). The polarization is directly related to the value of $\tan \beta$, as we discuss below.

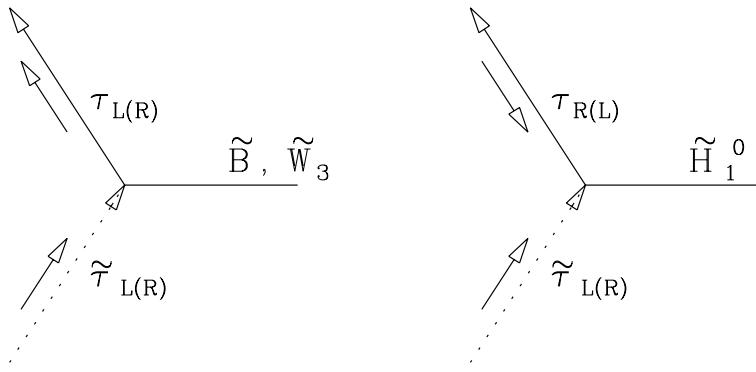


Figure 1:

Fig.1 shows the interaction of neutral components of gauginos and higgsinos to $\tilde{\tau}$ and τ . The interaction is completely fixed by the gauge and supersymmetry of the model. The coupling to the gaugino $\tilde{B}(\tilde{W}_3)$ is proportional to the gauge coupling $g_{1(2)}$, while the coupling of the $\tilde{\tau}$ to the Higgino \tilde{H}_1^0 is proportional to Y_τ . The two interactions are different not only in the couplings, but also in the chirality of the (s)fermion. The (super-) gauge interaction is chirality conserving, while the (super-) Yukawa interaction flips the chirality. (In the figure, the arrows

next to the $\tilde{\tau}$ and τ line show the direction of chirality.) Thus P_τ depends on the ratio of the chirality flipping and the conserving interactions.

As we mentioned already, gauginos and higgsinos are not mass eigenstates, but they mix to form neutralino mass eigenstates χ_i^0 . $\tilde{\tau}_R$ and $\tilde{\tau}_L$ also mix, thus the coupling of the $\tilde{\tau}_1 \tau \chi_i^0$ interaction depends on both the stau mixing θ_τ and the neutralino mixing N_{ij} . However, θ_τ can be determined independently by measuring the production cross section of $\tilde{\tau}$ precisely, as $\sigma(e^+e^- \rightarrow \tilde{\tau}^+\tilde{\tau}^-)$ depends on θ_τ only.*

For an illustrative purpose, let's discuss the case where $\tilde{\tau}_1 = \tilde{\tau}_R$. Then $P_\tau(\tilde{\tau}_R \rightarrow \chi_i^0 \tau)$ is expressed, in the limit that the final state τ is relativistic, as follows;

$$P_\tau(\tilde{\tau}_R \rightarrow \chi_i^0 \tau) = \frac{(\sqrt{2}gN_{11} \tan\theta_W)^2 - (Y_\tau N_{13})^2}{(\sqrt{2}gN_{11} \tan\theta_W)^2 + (Y_\tau N_{13})^2}. \quad (6)$$

Here N_{ij} is the neutralino diagonalization matrix appearing in Eq.(3) and Y_τ is the tau Yukawa coupling in Eq.(1).

In fig. 2 we show contours of constant $P_\tau(\tilde{\tau}_R \rightarrow \chi_1^0 \tau)$ in the $M_1 - \mu$ plane for $\tan\beta = 10$ (Fig. 2a) and $\tan\beta = 2$ (Fig. 2b). P_τ decreases monotonically as M_1 increases for a fixed value of μ , as χ_1^0 is bino-like ($N_{11} \simeq 1$) for $M_1 \ll |\mu|$, while χ_1^0 is higgsino-like if $M_1 \gg |\mu|$ ($N_{11} \ll 1$). One should also notice the strong dependence of P_τ on $\tan\beta$. This is because the Yukawa coupling of the τ lepton increases linearly with $\tan\beta$ when $\tan\beta$ is large.

We learned that the measurement of P_τ gives us a constraint to a combination of neutralino mixing N_{ij} and $\tan\beta$. Other sparticle productions also carry information about the neutralino sector. However as $\tan\beta$ becomes larger, the neutralino and chargino mass and mixing matrix become less and less dependent on $\tan\beta$. This is because the off-diagonal elements of the mass matrices of the neutralinos and charginos become insensitive to $\tan\beta$ once $\tan\beta > 10$, as $\cos\beta \sim 0$ and $\sin\beta \sim 1$.

To demonstrate this, we show various quantities in Fig.4 a)-d) fixing $m_{\chi_1^0} = 100$ GeV, and varying M_1 and $\tan\beta$. The reasons to take such a parametrization are following: According to the investigation of the production of $e\tilde{e}^+ \tilde{e}^- (\tilde{\mu}_R^+ \tilde{\mu}_R^-)$ and its subsequent decay to $e^\pm(\mu^\pm)\chi_1^0$ in Ref[3], one can determine the neutralino mass with an error of a few percent from the distribution of the final state leptons. The same analysis is also possible for the chargino pair production and decay, so it would be almost certain that we get some idea of the lightest neutralino mass $m_{\chi_1^0}$ once SUSY particle productions are observed at an e^+e^- collider. The corresponding curves that satisfy the constraint were shown in Fig. 3 on $M_1 - \mu$ plain, assuming $m_{\chi_1^0} = 100$ GeV (no error). Here the solid curve corresponds to $\tan\beta = 1.5$ and the dotted curve

*The measurement of σ is actually correlated to the mixing of neutralino, as the detection efficiency depends on the decay modes into $\chi_i^0 \tau$, which should be studied carefully. The detailed discussion can be found in [6]

$$P_\tau(\tilde{\tau}_R \rightarrow \chi_1^0 \tau)$$

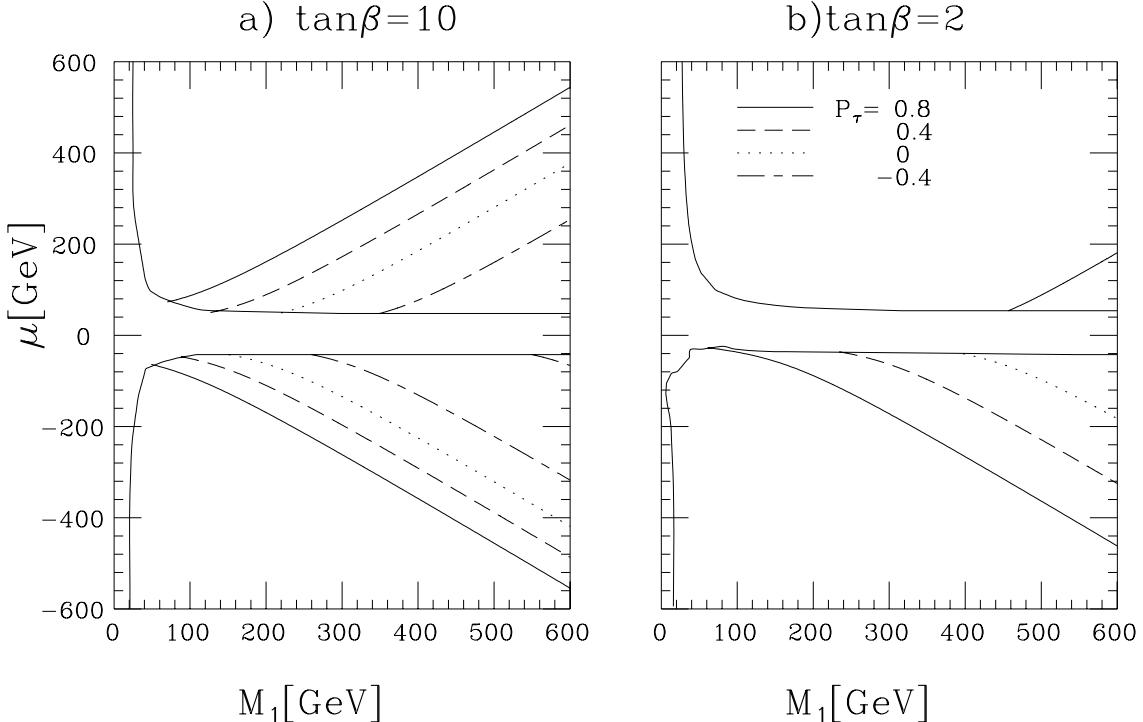


Figure 2:

corresponds to $\tan\beta = 30$.[†] With the mass constraint, one can specify the parameter space of the neutralino sector by M_1 and $\tan\beta$, up to twofold ambiguity of positive and negative μ solutions for the large M_1 and $\tan\beta$ region, or up to threefold ambiguity (2 solutions in the negative μ region and one solution in the positive μ region) for the small values of M_1 and $\tan\beta$. We take the positive μ solution throughout the plots Fig. 4 a)-d) to avoid too many lines (sometimes quite close each other) appearing on the same plot.[‡]

Fig. 4a and fig. 4b show the contours of the mass differences a) $m_{\chi_1^+} - m_{\chi_1^0}$ and b) $m_{\chi_2^0} - m_{\chi_1^0}$. One can see the mass difference depends very mildly on $\tan\beta$ once $\tan\beta > 5$. The tendency is also same for the negative μ solution, though the mass differences decreases as $\tan\beta$ becomes smaller for the negative μ solution. The dependence on $\tan\beta$ is even milder for $\sigma(e^+e^- \rightarrow \tilde{e}_R^+\tilde{e}_R^-)$

[†]We assumed GUT relation to the gaugino masses

[‡]The ambiguity of μ might be removed by other experiment, such as $Br(b \rightarrow s\gamma)$ [10]

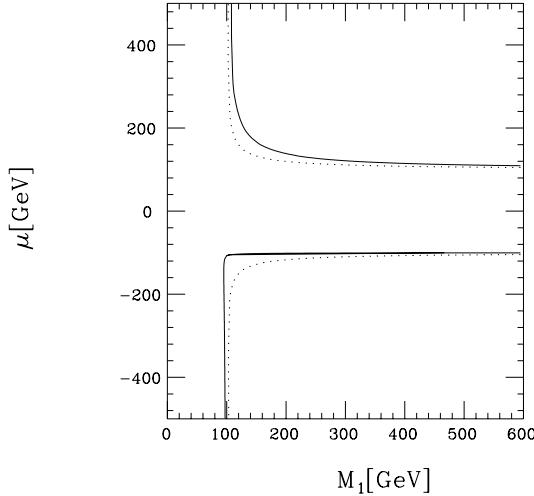


Figure 3:

as can be seen in Fig 4c. The production proceeds through the s-channel exchange of gauge bosons and the t-channel exchange of neutralinos, where the dependence on M_1 comes in. The \tilde{e}_R production cross section turns out to be the best quantity to fix M_1 , free from the uncertainty of the value of $\tan\beta$.

Fig. 4d) is the contour plot of constant P_τ ($\tilde{\tau}_R \rightarrow \chi_1^0 \tau$). The plot looks totally different from Fig 4a-c). If M_1 is not too close to 100 GeV, the polarization depends on $\tan\beta$ sensitively for the parameters shown in the figure. If one knows M_1 precisely from the production cross section of \tilde{e}_R or from other processes, one can extract $\tan\beta$ by further measuring $P_\tau(\tilde{\tau}_R \rightarrow \chi_1^0 \tau)$. Notice that the sensitivity is better in the region $\tan\beta > 5$, complimentary to the information from the mass differences of -inos. (See Fig 4a and 4b).

For most of the parameter space, $m_{\chi_1^0}$, $m_{\chi_2^0}$, $m_{\chi_1^+}$ are very close to each other, thus the decay mode into those -inos are always open. The determination of the branching ratios constrain the model parameters even further.

Notice the decay into the lightest neutralino may not be the dominant decay mode. If the decay modes into the gaugino like -ino are open, $\tilde{\tau}$ decays dominantly into it, even if the higgsino-like -ino is lighter than the gaugino-like -ino. As the gaugino coupling is insensitive to $\tan\beta$, we will not be able to determine $\tan\beta$ in such a case.

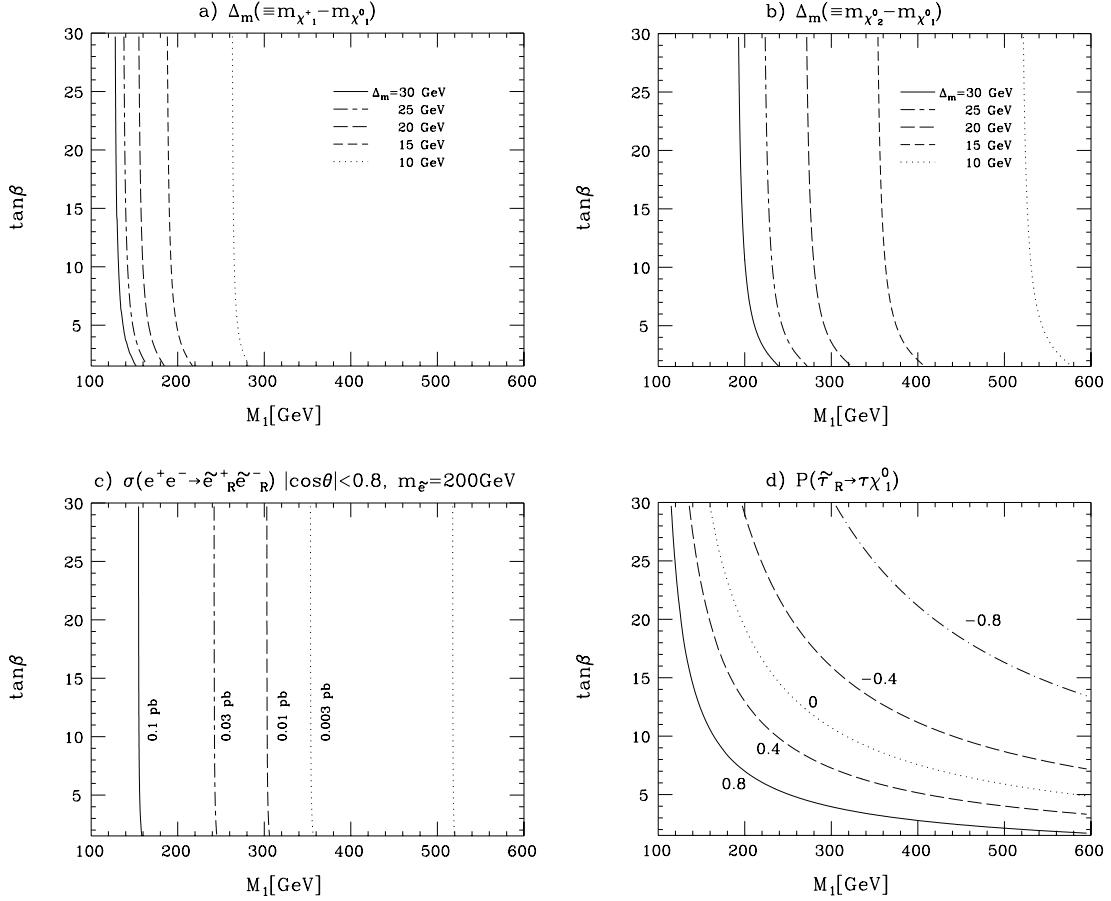


Figure 4:

4) Measurement of P_τ

The measurement of P_τ would be carried out through the energy distribution of decay products from the polarized τ lepton. The τ lepton decays into $A\nu_\tau$ where $A = e\nu, \pi, \rho, a_1, \dots$. The decay distributions of the τ decay products depend on the polarization of the parent τ lepton[10]. In particular, for each decay channel the momentum distribution of the decay products ($\pi^-, \rho \rightarrow \pi^-\pi^0, \dots$) differs significantly depending on whether their parent is $\tau_R^- (h = 1/2)$ or $\tau_L^- (h = -1/2)$. If the τ lepton is relativistic, P_τ can then be determined from the energy distribution of the decay products [8]. Notice that the τ lepton from a $\tilde{\tau}$ decay also has some energy distribution which depends on $m_{\tilde{\tau}_1}$ and $m_{\chi_1^0}$, thus the energy spectrum of the final decay products depends

on $m_{\tilde{\tau}_1}$, $m_{\chi_1^0}$ and P_τ . The three quantity, in principle, can be determined from the energy distribution only by fitting the energy spectrum, but this is experimentally rather challenging. The situation can be improved by using information from other sources: $m_{\chi_1^0}$ from, for instance, selectron production and decay, and $m_{\tilde{\tau}_1}$ from a threshold scan.

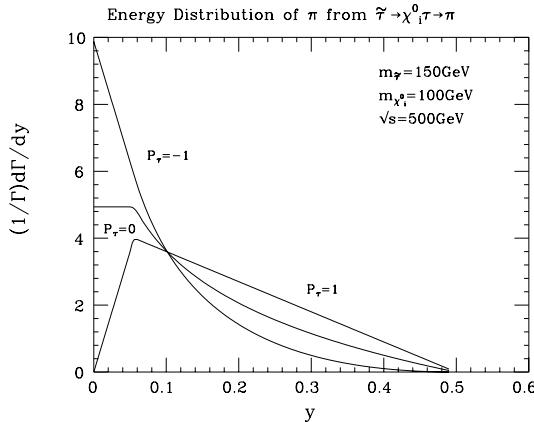


Figure 5:

In fig. 5 we have shown the normalized energy distribution of the π ($y = E_\pi/E_{\text{beam}}$) from the cascade decay of $\tilde{\tau}_1$. For the plot we took $P_\tau = \pm 1, 0$, $m_{\tilde{\tau}_1} = 150$ GeV, $m_{\chi_1^0} = 100$ GeV and $\sqrt{s} = 500$ GeV. The spectrum is considerably harder (softer) for $\tau_{R(L)}$. The upper end of the energy distribution (y_{max}) is the maximum energy of the τ lepton arising from $\tilde{\tau}$, while the minimum energy of the τ lepton corresponds to the peak of the energy distribution for $P_\tau = 1$ (y_{min}). For the parameter used in Fig 5, 52%(90%) of the events are in $y > y_{\text{min}}$ region for $P_\tau = -1(1)$, while 4% (22%) of the events go above $(y_{\text{min}} + y_{\text{max}})/2$.

P_τ can also be measured independently by studying the distributions of the difference of the energy between decay pions from ρ and a_1 . The $\tau \rightarrow \rho\nu$ mode has a branching ratio of about 23% and the $\tau \rightarrow a_1\nu$ mode has a branching ratio of about 15%. $\tau_{L(R)}$ decays dominantly into the longitudinal (transverse) element of ρ or a_1 , and they tend to get most of the τ energy. Then, a transversely polarized ρ favors equal splitting of the ρ energy between the two decay pions, whereas a longitudinally polarized ρ leads to a large difference of the π^- and π^0 energies. For a_{1T} , all three pions have a tendency to share equally the energy of a_1 . On the other hand, a_{1L} again favors configurations in which one or two of the pions are soft. A detailed discussion of the energy distribution may be found in [8].

Monte Carlo simulations of the determination of the τ lepton polarization are in progress [11]. For this purpose, we have developed a Monte Carlo event generator of the signal pro-

cess ($e^+e^- \rightarrow \tilde{\tau}^+\tilde{\tau}^-$). The generator takes into account the polarization of decaying taus, by using the TAUOLA2.4 program package[9]. This package is implemented together with the JETSET7.3 program in a single program module dealing with the hadronization step and is being used for both the signal and background processes. The generated events are processed through the standard JLC detector simulator whose parameters can be found in [12].

As an example to demonstrate the effects of the tau polarization in the stau decays, we show in Fig.6 the momentum fraction distributions of π 's from a_1 decays after a set of cuts to select 1-3 topology. We took $m_{\tilde{\tau}} = 141.9$ GeV, $m_{\chi_1^0} = 117.8$ GeV and compare the energy distribution for $P_\tau = 1$ and 0.

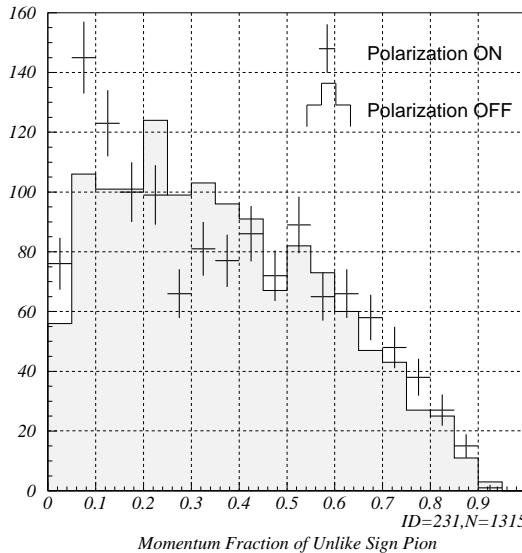


Figure 6:

The original sample contains 5k $\tilde{\tau}$ -pair events in which one tau was forced to decay into $\pi\nu_\tau$ and the other into $a_1\nu_\tau$. The background processes such as W^+W^- , ZZ , ZH , $\gamma\gamma$, etc have not been included yet.

Studies of the other decay modes are ongoing together with the background processes.

In summary, the measurement of $P_\tau(\tilde{\tau} \rightarrow \tau\chi_i^0)$ could give unique information about $\tan\beta$, combined with the the information from other measurements such as $m_{\chi_2^0} - m_{\chi_1^0}$ or $\sigma(e^+e^- \rightarrow \tilde{e}^+\tilde{e}^-)$. In some sense Fig. 4 might be regarded as key maps of the parameter space of MSSM. After years of running of JLC, we may put our finger on a point of the parameter space by the precise measurement of event signatures of sparticle productions.

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